

# Black-Box Optimization Benchmarking Template for the Comparison of More than Two Algorithms on the Noiseless Testbed

Draft version \*

Forename Name

## ABSTRACT

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

Benchmarking, Black-box optimization

## 1. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number of trials that actually reached  $f_t$  [?, ?]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  ( $10^{-8}$  as in Figure 1) using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

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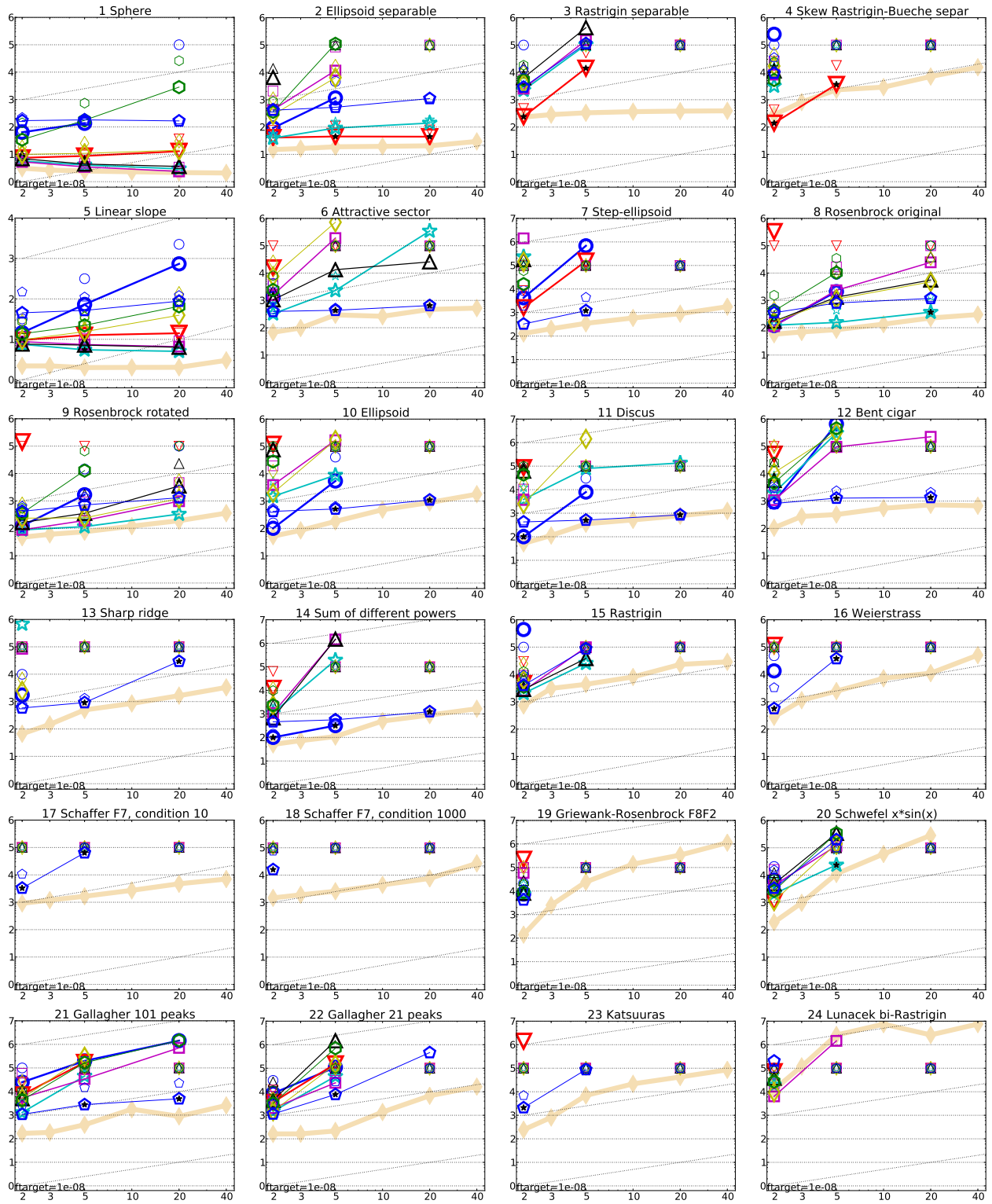


Figure 1: Expected running time (ERT in number of  $f$ -evaluations) divided by dimension for target function value  $10^{-8}$  as  $\log_{10}$  values versus dimension. Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :Nelder-Mead,  $\nabla$ :Powell,  $\star$ :BFGS,  $\square$ :L-BFGS-B,  $\triangle$ :CG,  $\diamond$ :SLSQP,  $\circ$ :TNC,  $\pentagon$ :CMA

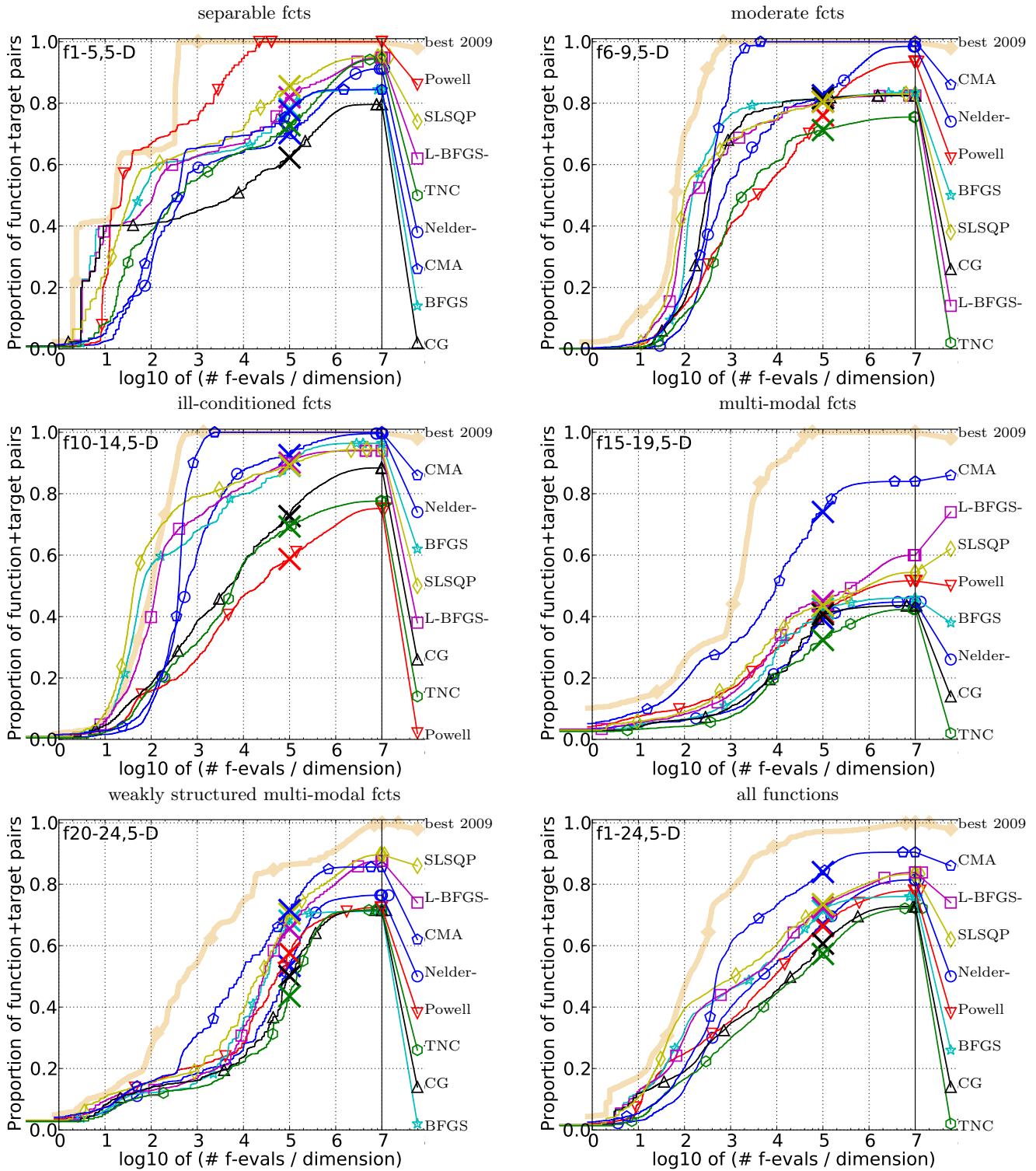


Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in  $10^{[-8..2]}$  for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

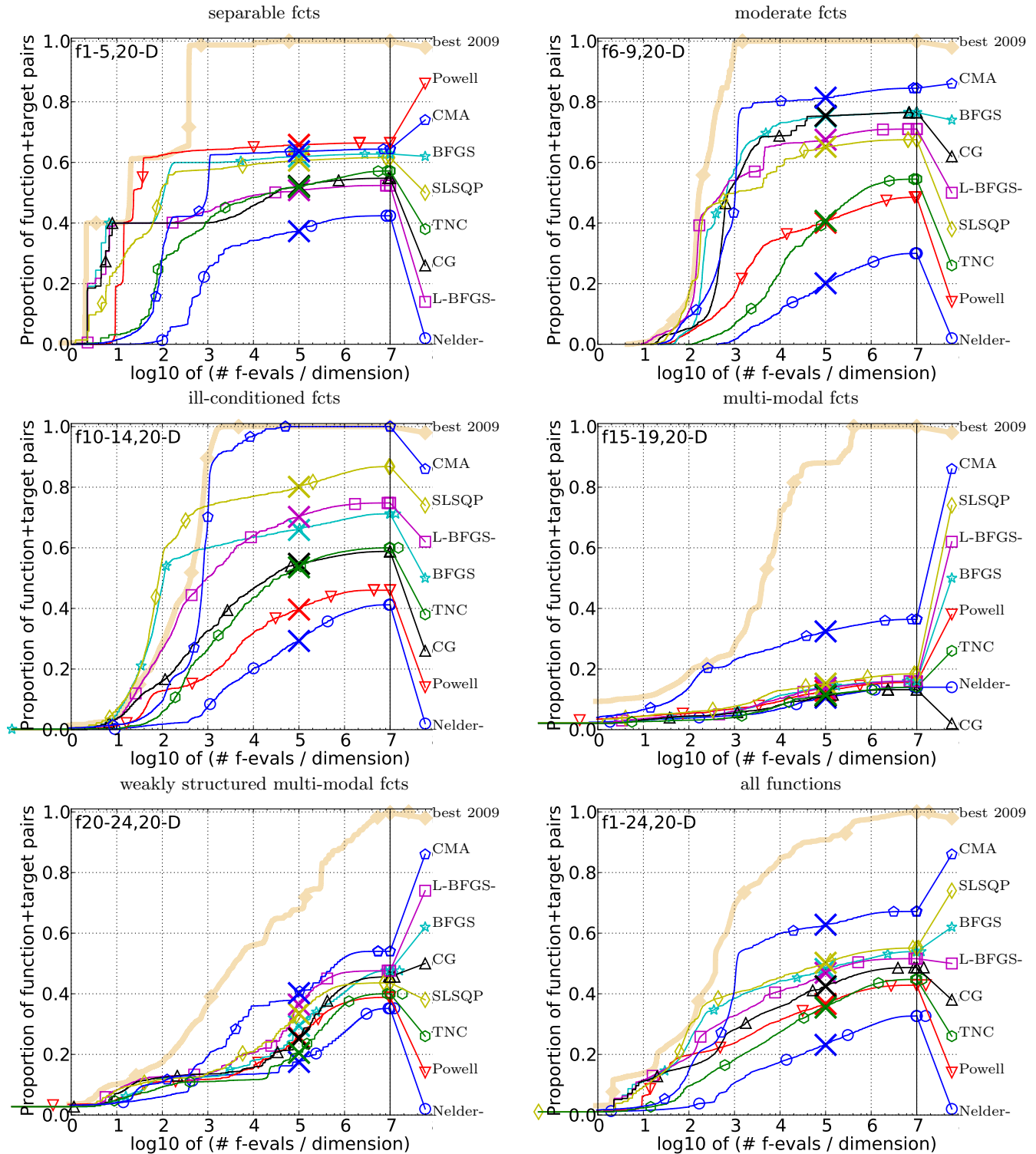


Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in  $10^{[-8..2]}$  for all functions and subgroups in 20-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f1</b>	11	12	12	12	12	12	15/15 <b>f13</b>	132	195	250	1310	1752	2255	15/15
Nelder-Powell	12(13)	27(11)	34(8)	40(8)	47(10)	52(10)	15/15 Nelder-Powell	9.3(11)	8.8(9)	8.9(8)	6.9(10)	<b>66(85)</b>	<b>3110(3271)</b>	0/15
BFGS	3.1(1)	3.4(0.2)	3.5(0.2)	3.6(0.2)	3.6(0.2)	3.6(0.2)	15/15 BFGS	26(35)	35(38)	37(31)	165(217)	4202(4711)	$\infty$ 5e5	0/15
L-BFGS-CG	1.4(0)	1.2(0)	1.2(0)	1.2(0)	1.2(0)	1.7(0.3)	15/15 L-BFGS-CG	1.0(0.1)	1.0(0.1)	1.00(0.1)	1.0(0.8)	464(571)	$\infty$ 5e5	0/15
SLSQP	1.4(0.0)	1.3(0.0)	1.3(0.0)	1.4(0.3)	1.4(0.3)	1.4(0.3)	15/15 SLSQP	6.4(9)	22(28)	23(25)	16(20)	91(118)	$\infty$ 5e5	0/15
TNC	1.4(0)	1.2(0)	1.2(0)	1.2(0)	1.2(0)	1.8(0.6)	15/15 TNC	16(40)	28(32)	30(26)	20(22)	209(222)	$\infty$ 5e5	0/15
CMA	2.8(1)	5.1(4)	8.6(8)	18(13)	35(37)	58(71)	15/15 CMA	<b>0.78(0.1)</b>	<b>*0.80(0.1)*</b>	<b>*0.82(0.1)*</b>	5.9(2)	1186(1427)	$\infty$ 5e5	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f2</b>	83	87	88	90	92	94	15/15 <b>f14</b>	10	41	58	139	251	476	15/15
Nelder-Powell	23(11)	26(14)	28(14)	29(14)	33(21)	40(38)	15/15 Nelder-Powell	16(14)	10(4)	8.8(2)	5.1(7)	4.0(0.8)	<b>2.7(0.4)*</b>	15/15
BFGS	1.1(0.4)*	1.2(0.4)*	1.3(0.1)*	1.4(0.1)*	1.9(0.4)	2.1(0.2)*	15/15 BFGS	4.6(3)	3.0(1)	2.7(0.7)	1181(2063)	$\infty$ 5e5	$\infty$ 5e5	0/15
L-BFGS-CG	2.7(1)	2.9(1)	3.3(2)	3.9(3)	4.2(3)	4.6(3)	15/15 L-BFGS-CG	1.9(0.8)	0.93(0.4)	1.2(0.3)	1.1(0.1)	0.93(0.1)	85(90)	6/15
SLSQP	3.2(2)	4.1(2)	5.1(2)	7.7(3)	8.2(3)	44(40)	15/15 SLSQP	2.2(0.8)	1.4(0.5)	1.5(0.3)	1.2(0.2)	1.6(1)	685(898)	1/15
TNC	64(67)	222(168)	420(303)	2522(2032)	2.5e4(3e4)	$\infty$ 5e5	15/15 TNC	2.2(2)	1.2(0.5)	1.4(0.5)	1.8(0.4)	7.8(8)	4959(5255)	1/15
CMA	1.5(0.6)	1.7(0.6)	1.8(0.7)	2.2(0.8)	2.5(0.9)	11(11)	15/15 CMA	1.3(0.9)	0.79(0.4)	1.0(0.4)	0.87(0.2)*	<b>0.76(0.1)*</b>	<b>*37(1110)</b>	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f3</b>	716	1622	1637	1646	1650	1654	15/15 <b>f15</b>	511	9310	19369	20073	20769	21359	14/15
Nelder-Powell	144(132)	632(668)	1403(1520)	1396(1519)	1392(1507)	$\infty$ 5e5	15/15 Nelder-Powell	34(25)	5.2(4)	11(10)	11(9)	11(9)	$\infty$ 5e5	0/15
BFGS	0.57(1)*	25(27)	25(27)	25(27)	26(25)*	26(25)*	15/15 BFGS	12(11)	5.9(5)	10(11)	11(10)	102(108)	$\infty$ 5e5	0/15
L-BFGS-CG	27(25)	86(122)	311(313)	309(302)	308(303)	308(335)	15/15 L-BFGS-CG	15(11)	3.0(2)	6.6(9)	6.4(9)	6.2(8)	6.0(8)	5/15
SLSQP	13(16)	55(84)	176(198)	175(156)	174(156)	217(227)	15/15 SLSQP	6.4(5)	2.0(2)	2.8(2)	2.7(2)	2.6(2)	4.9(4)	9/15
TNC	71(78)	348(375)	1316(1516)	1309(1508)	1305(1468)	1303(1512)	15/15 TNC	25(15)	4.1(4)	10(9)	9.3(9)	9.0(8)	8.7(8)	15/15
CMA	3.3(3)	10(11)	56(67)	55(66)	55(66)	267(211)	15/15 CMA	3.2(3)	0.92(0.8)	1.5(2)	1.4(2)	1.7(2)	19(23)	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f4</b>	809	1633	1688	1817	1886	1903	15/15 <b>f16</b>	120	612	2662	10449	11644	12095	15/15
Nelder-Powell	161(128)	474(512)	4178(4295)	3881(4540)	3740(4110)	$\infty$ 5e5	15/15 Nelder-Powell	25(39)	57(68)	64(57)	$\infty$	$\infty$	$\infty$ 5e5	0/15
BFGS	0.23(0.3)*	0.86(0.6)*	5.1(6)*	4.8(6)*	4.6(5)*	5.0(5)*	15/15 BFGS	10(16)	17(15)	20(18)	679(742)	$\infty$ 5e5	$\infty$ 5e5	0/15
L-BFGS-CG	55(37)	4459(5204)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	15/15 L-BFGS-CG	250(240)	1760(1808)	$\infty$	$\infty$	$\infty$ 5e5	$\infty$ 5e5	0/15
SLSQP	28(32)	536(503)	4369(4295)	4058(4334)	3911(4640)	$\infty$ 5e5	15/15 SLSQP	35(38)	310(354)	2801(3005)	$\infty$	$\infty$	$\infty$ 5e5	0/15
TNC	257(340)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	15/15 TNC	91(96)	2023(2099)	$\infty$	$\infty$	$\infty$ 5e5	$\infty$ 5e5	0/15
CMA	8.9(12)	93(78)	1003(1102)	932(1023)	3748(4242)	$\infty$ 5e5	15/15 CMA	19(24)	218(198)	2642(3005)	$\infty$	$\infty$	$\infty$ 5e5	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f5</b>	10	10	10	10	10	10	15/15 <b>f17</b>	5.2	215	899	3669	6351	7934	15/15
Nelder-Powell	25(38)	35(44)	36(44)	36(44)	36(44)	36(44)	15/15 Nelder-Powell	2211(2793)	2749(2942)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	0/15
BFGS	6.2(0.5)	6.2(0.5)	6.3(0.5)	6.3(0.5)	6.3(0.5)	6.3(0.5)	15/15 BFGS	120(153)	92(83)	48(55)	$\infty$	$\infty$	$\infty$ 5e5	0/15
L-BFGS-CG	1.7(0.8)	2.6(0.8)	2.7(0.8)	2.8(0.8)	2.8(0.8)	2.8(0.8)	15/15 L-BFGS-CG	101(237)	67(54)	1348(1497)	$\infty$	$\infty$	$\infty$ 5e5	0/15
SLSQP	1.7(0.3)	3.5(2)	3.6(2)	3.7(2)	3.7(2)	3.7(2)	15/15 SLSQP	36(34)	45(52)	244(294)	$\infty$	$\infty$	$\infty$ 5e5	0/15
TNC	1.9(1)	3.4(1)	3.6(1)	3.6(1)	3.6(1)	3.6(1)	15/15 TNC	508(828)	221(272)	1846(1771)	$\infty$	$\infty$	$\infty$ 5e5	0/15
CMA	1.8(1)	4.9(3)	6.6(4)	7.6(5)	7.7(5)	7.7(5)	15/15 CMA	32(37)	21(10)	421(390)	$\infty$	$\infty$	$\infty$ 5e5	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f6</b>	114	214	261	580	1038	1332	15/15 <b>f18</b>	103	378	3968	9280	10905	12469	15/15
Nelder-Powell	14(11)	13(10)	13(9)	13(10)	23(27)	1151(1311)	15/15 Nelder-Powell	566(392)	1797(1573)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	0/15
BFGS	7.0(7)	7.0(7)	8.8(8)	112(113)	2055(2359)	5347(6008)	15/15 BFGS	30(31)	48(46)	118(114)	$\infty$	$\infty$	$\infty$ 5e5	0/15
L-BFGS-CG	5.5(4)	4.1(2)	4.0(2)	3.1(2)	2.6(2)	4.1(4)	15/15 L-BFGS-CG	55(57)	396(508)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	0/15
SLSQP	4.3(3)	4.0(2)	4.2(2)	2.8(1)	4.6(4)	191(186)	15/15 SLSQP	72(163)	212(288)	1775(2079)	$\infty$	$\infty$	$\infty$ 5e5	0/15
TNC	7.1(6)	6.5(4)	7.2(4)	5.3(2)	4.7(3)	6.9(6)	15/15 TNC	590(394)	2206(2064)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	0/15
CMA	1.8(2)	1.6(0.8)	1.8(1.0)	2.9(3)	7.7(9)	288(281)	15/15 CMA	32(36)	98(55)	1813(1764)	$\infty$	$\infty$	$\infty$ 5e5	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f7</b>	24	324	1171	1572	1572	1597	15/15 <b>f19</b>	1	1	242	1.2e5	1.2e5	1.2e5	15/15
Nelder-Powell	461(398)	99(152)	306(338)	2178(2442)	2178(2124)	2145(2561)	15/15 Nelder-Powell	4374(4898)	2.1e4(2e4)	1281(1460)	$\infty$	$\infty$	$\infty$ 5e5	0/15
BFGS	33(27)	14(19)	14(16)	291(297)	291(274)	463(518)	15/15 BFGS	183(358)	7666(8482)	710(680)	$\infty$	$\infty$	$\infty$ 5e5	0/15
L-BFGS-CG	178(331)	210(175)	6231(6618)	$\infty$	$\infty$	$\infty$ 5e5	15/15 L-BFGS-CG	3546(4608)	1.4e4(9298)	744(694)	$\infty$	$\infty$	$\infty$ 5e5	0/15
SLSQP	197(302)	223(370)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	15/15 SLSQP	2848(4106)	1.1e4(5890)	165(166)	59(64)	59(66)	58(61)	0/15
TNC	308(539)	242(296)	$\infty$	$\infty$	$\infty$	$\infty$ 5e5	15/15 TNC	4600(4484)	2.7e4(2e4)	1107(1115)	$\infty$	$\infty$	$\infty$ 5e5	0/15
CMA	201(213)	107(140)	6087(6190)	$\infty$	$\infty$	$\infty$ 5e5	15/15 CMA	4345(6048)	2.2e4(2e4)	441(462)	60(64)	59(67)	$\infty$ 5e5	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f8</b>	73	273	336	391	410	422	15/15 <b>f20</b>	16	851	38111	54470	54861	55313	14/15
Nelder-Powell	6.5(6)	33(79)	28(64)	24(55)	24(52)	24(51)	15/15 Nelder-Powell	19(18)	11(16)	6.6(8)	4.6(6)	4.6(6)	5.4(6)	0/15
BFGS	3.2(3)	18(40)	30(43)	42(49)	330(389)	8283(8859)	15/15 BFGS	8.3(3)	1.5(1)	3.1(3)	3.1(4)	23(27)	129(149)	0/15
L-BFGS-CG	3.2(1)	2.2(2)	2.0(1)	1.9(1)	1.9(1)	1.8(1)	15/15 L-BFGS-CG	6.9(4)	4.0(4)	3.1(4)	2.2(3)	2.1(3)	2.1(3)	15/15
SLSQP	1.9(0.9)	42(120)	34(97)	29(84)	28(80)	27(77)	15/15 SLSQP	1.8(0.8)	6.1(5)	16(17)	11(11)	11(11)	11(11)	9/15
TNC	2.7(2)	20(16)	17(13)	16(11)	15(10)	15(10)	15/15 TNC	2.4(0.7)	33(43)	44(47)	31(32)	31(32)	30(28)	4/15
CMA	1.6(0.6)	17(16)	14(13)	12(11)	11(11)	12(10)	15/15 CMA	1.5(1)	3.6(4)	10(10)	7.0(7)	7.0(7)	9.2(9)	8/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ $\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f9</b>	116	127	214	300	335	369	15/15 <b>f21</b>	41	1157	1674	1705	1729	1757	14/15
Nelder-Powell	11(6)	60(81)	37(48)	27(35)	24(31)	22(28)	15/15 Nelder-Powell	693(1002)	469(365)	593(652)	582(643)	575(596)	565(586)	6/15
BFGS	6.9(5)	30(15)	20(9)	38(61)	360(314)	$\infty$ 5e5	15/15 BFGS	50(2)	130(162)	170(189)				

$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f1</b>	43	43	43	43	43	43	15/15	<b>f13</b>	652	2021	2751	18749	24455	30201	15/15
Nelder-Powell	178(50)	288(90)	393(84)	1383(1512)	3.8e4(5e4)	8.5e4(1e5)	0/15	Nelder-Powell	137(124)	240(495)	807(916)	$\infty$	$\infty$	$\infty$	0/15
BFGS	4.2(0.0)	4.3(0.1)	4.3(0.0)	4.3(0.0)	5.2(0.0)	6.0(6)	15/15	BFGS	24(27)	23(20)	24(16)	706(846)	$\infty$	$\infty$	0/15
L-BFGS-CG	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.4(0.3)	15/15	L-BFGS-CG	1.5(0.1)	0.67(0.0)	0.62(0.0)	0.03(144)	1145(1329)	$\infty$	0/15
SLSQP	1.1(0)	1.1(0)	1.1(0)	1.1(0)	1.1(0)	1.1(0)	0/15	SLSQP	6.9(10)	7.7(8)	13(15)	6.9(5)	122(141)	$\infty$	0/15
TNC	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.7(0.5)	15/15	TNC	5.5(7)	6.5(8)	6.8(8)	3.2(3)	46(42)	$\infty$	0/15
CMA	10(1)	18(3)	25(2)	40(4)	55(4)	70(5)	15/15	CMA	1.2(0.1)	0.56(0.0)	0.55(0.0)	0.34(54)	$\infty$	0/15	
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f2</b>	385	386	387	390	391	393	15/15	<b>f14</b>	75	239	304	932	1648	15661	15/15
Nelder-Powell	7.5e4(8e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	Nelder-Powell	64(40)	62(28)	78(29)	1067(1200)	$\infty$	$\infty$	0/15
BFGS	1.2(0.0)	1.2(0.0)	1.2(0.0)	1.4(0.4)	1.9(0.1)	1.9(0.0)	15/15	BFGS	4.9(1)	2.7(0.5)	2.9(0.5)	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	4.5(1)	4.9(2)	5.2(1)	5.8(1)	6.3(1)	6.8(1)	0/15	L-BFGS-CG	1.6(0.3)	0.91(0.2)	1.1(0.3)	0.79(0.1)	0.73(0.1)	$\infty$	0/15
SLSQP	26(10)	70(45)	159(137)	$\infty$	$\infty$	$\infty$	0/15	SLSQP	1.8(0.3)	0.96(0.2)	1.1(0.2)	0.97(0.1)	1463(1471)	$\infty$	0/15
TNC	206(66)	416(238)	864(474)	3688(3548)	$\infty$	$\infty$	0/15	TNC	1.7(0.4)	0.99(0.2)	1.2(0.3)	2.3(0.6)	20(13)	$\infty$	0/15
CMA	3.2(0.8)	3.8(0.7)	4.1(0.7)	4.4(0.9)	4.8(0.8)	1136(797)	15/15	CMA	1.1(0.2)	0.68(0.1)	0.85(0.1)	0.61(0.1)	0.70(0.3)	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f3</b>	5066	7626	7635	7643	7646	7651	15/15	<b>f15</b>	1812	14(9)	21(10)	3.2e4(3e4)	7.0(0.7)	1.3(0.1)	15/15
Nelder-Powell	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	Nelder-Powell	30378	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
BFGS	11(9)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	BFGS	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	L-BFGS-CG	206(230)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
SLSQP	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	SLSQP	928(988)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
TNC	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	TNC	218(249)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
CMA	1821(2031)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	CMA	108(113)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f4</b>	4722	7628	7666	7700	7758	1.4e5	9/15	<b>f16</b>	1384	27265	77015	1.9e5	2.0e5	2.2e5	15/15
Nelder-Powell	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	Nelder-Powell	170(184)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
BFGS	14(18)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	BFGS	40(49)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	L-BFGS-CG	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
SLSQP	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	SLSQP	4900(5327)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
TNC	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	TNC	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
CMA	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	CMA	1.7(0.9)	1.1(1)	48(55)	$\infty$	$\infty$	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f5</b>	41	41	41	41	41	41	15/15	<b>f17</b>	63	1030	4005	30677	56288	80472	15/15
Nelder-Powell	260(217)	359(390)	362(393)	363(394)	363(394)	363(394)	0/15	Nelder-Powell	5741(4836)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
BFGS	6.9(0.2)	7.0(0.1)	7.0(0.1)	7.0(0.1)	7.0(0.1)	7.0(0.1)	15/15	BFGS	821(1244)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	1.5(0.3)	2.4(0.6)	2.5(0.3)	2.5(0.3)	2.5(0.3)	2.5(0.3)	15/15	L-BFGS-CG	184(195)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
SLSQP	2.2(0)	3.1(0.6)	3.2(0.3)	3.2(0.3)	3.2(0.3)	3.2(0.3)	15/15	SLSQP	301(385)	1.4e4(2e4)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
TNC	2.2(0.6)	2.9(0.6)	3.1(0.8)	3.1(0.8)	3.1(0.8)	3.1(0.8)	15/15	TNC	4150(7383)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
CMA	2.2(0.3)	4.1(3)	14(31)	19(31)	19(31)	19(31)	15/15	CMA	111(208)	2.8e4(3e4)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f6</b>	1296	2343	3413	5220	6728	8409	15/15	<b>f18</b>	621	3972	19561	67569	1.3e5	1.5e5	15/15
Nelder-Powell	109(64)	560(436)	$\infty$	$\infty$	$\infty$	$\infty$	0/15	Nelder-Powell	4.6e4(5e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
BFGS	25(19)	23(15)	604(704)	$\infty$	$\infty$	$\infty$	0/15	BFGS	8402(8713)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	6.3(2)	5.0(2)	5.2(1)	5.9(3)	11(7)	22(19)	0/15	L-BFGS-CG	1314(1807)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
SLSQP	3.1(1)	3.1(0.6)	3.0(0.4)	4.7(2)	840(1046)	$\infty$	0/15	SLSQP	760(832)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
TNC	4.3(1)	4.2(1)	4.5(1)	4.8(2)	5.7(2)	7.1(6)	13/15	TNC	336(225)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
CMA	1.7(0.7)	2.1(2)	4.9(4)	120(112)	$\infty$	$\infty$	0/15	CMA	4220(4529)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f7</b>	1351	4274	9503	16524	16524	16969	15/15	<b>f19</b>	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
Nelder-Powell	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	Nelder-Powell	4.3e5(3e5)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
BFGS	62(26)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	BFGS	1.1e5(8e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	L-BFGS-CG	7.1e4(3e4)	2.8e7(3e7)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
SLSQP	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	SLSQP	4.5e5(5e5)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
TNC	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15	TNC	1.1e5(1e5)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
CMA	1.4(1)	117(128)	978(1052)	$\infty$	$\infty$	$\infty$	0/15	CMA	3.5e4(3e4)	7.6e5(6e5)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f8</b>	2039	3871	4040	4219	4371	4484	15/15	<b>f20</b>	82	46150	3.1e6	5.5e6	5.6e6	5.6e6	14/15
Nelder-Powell	42(41)	172(214)	447(530)	6842(7110)	$\infty$	$\infty$	0/15	Nelder-Powell	103(28)	196(205)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
BFGS	1.8(1)	4.1(5)	6.9(6)	1058(1195)	$\infty$	$\infty$	0/15	BFGS	10(2)	0.27(0.4)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
L-BFGS-CG	1.2(0.2)	1.8(1)	1.8(1)	1.7(1)	1.7(1)	1.6(1)	15/15	L-BFGS-CG	13(4)	14(12)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
SLSQP	0.58(0.2)	3.4(9.11)	4.8(11)	4.6(10)	4.5(10)	5.6(9)	13/15	SLSQP	1.6(0.4)	30(34)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
TNC	1.6(0.5)	27(91)	26(88)	26(84)	25(81)	25(79)	15/15	TNC	1.9(0.3)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
CMA	0.68(0.3)	22(32)	21(30)	20(29)	19(28)	19(27)	0/15	CMA	2.1(0.9)	308(344)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f_{opt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
<b>f9</b>	1716	3102	3277	3455	3594	3727	15/15	<b>f21</b>	561	6541	14103	14643	15567	17589	15/15
Nelder-Powell	245(231)	4478(5159)	$\infty$	$\infty$	$\infty$	$\infty$	0/15	Nelder-Powell	1004(1301)	1264(1528)	2037(2198)	1963(2185)	1848(1895)	1637(1	